Math 302 - Abstract Algebra

Fall 2025

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Office Hours: Office Hours: M 1:15-2:45pm, Tu 1-2pm, Th 11:15am-12:30pm, and by appointment

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Course Description

Abstract algebra is a beautiful branch of mathematics that, in addition to being worthy of study in its own right, provides useful tools to all other areas of mathematics. One characterization of modern algebra is that it is the abstraction of familiar arithmetic properties of number systems such as the integers or the reals to a more general setting: that of *groups*, *rings*, and *fields*. Another characterization is that it is the study of symmetry. By using a group to represent the symmetries of an object, we can often see much more clearly the essence of the question we are trying to answer.

We will spend roughly the first two-thirds of this course studying our most basic algebraic objects: groups. We will begin by learning what a group is and considering several examples of groups. Then we will study properties of groups and functions that map between them. We will end our study of groups with factor groups and the first isomorphism theorem. After we have gotten used to working with abstract objects, we will move on to a study of rings and fields. Time permitting, at the end of the course we will study some mathematical applications of the material we have learned.

Course Website

The website for this course can be found at:

http://f25.middlebury.edu/MATH0302A

Here you will find course information and homework assignments.

Text

We will cover most of Chapters 0-15 from *Contemporary Abstract Algebra* by Joseph A. Gallian. This textbook is currently in the 11th edition but you do not need to get the most recent edition of the book. You can safely use the 7th, 8th, 9th, 10th, or 11th edition.

Course Objectives

The goals of this course are to help you develop:

- an understanding of the concepts, notation, and theory of abstract algebra,
- the ability to use your understanding to solve straightforward and complex problems,
- proof-writing skills,
- skills in collaboration, and
- the confidence that comes with taking ownership of your learning process.

Homework

Homework assignments will be posted on the course website. Homework will be assigned weekly. The homework for each week (Monday, Wednesday, and Friday) will be collected at the beginning of class on the following Wednesday.

Many students find abstract algebra to be quite different from math courses they have previously taken. In order to learn and develop intuition for abstract algebra, it is essential that you work carefully through each homework assignment. Each assignment will involve a number of proofs so that you can also work on your mathematical writing skills. When writing, aim for clean, logically-ordered, and clearly-written arguments that would be understandable to your peers.

Along with working problems, reading the book and your notes is an important component of your homework. We will not necessarily cover every word of the chapters that we consider in class, but you should read them nonetheless. Be an active reader: read with a paper and pencil. Work out details as you read and frequently ask yourself such questions as "Where was the hypothesis used in this proof?" and "Do I understand each individual line in this proof as well as the general flow of the overall argument?" Reading will feel slow (especially compared to reading a novel) and often frustrating. That is normal. The payoff is that it can be deeply satisfying to understand something if you've had to work at it for a while first. Plus, the book will provide you with many examples of proofs after which you can model your own.

Homework exercises will be marked on a 0-5 scale. A 5 is given for a clear and consistent answer, scores of 1-4 are given for answers that have some logical flaws or that need more explanation, and a score of 0 is given when no attempt is made to answer the problem. Notice that homework is a relatively large percentage of your final grade so this should be where a significant amount of your energy for the class goes.

Note that **no late homework will be accepted**. Late homework will receive a score of zero. You may, however, hand in homework early if a conflict arises. Given that life often happens in ways that are out of our control, I will drop your lowest homework score when calculating your final grade. If you find yourself in a situation that warrants some more leeway, please be in touch so that we can talk about it.

Exams

There will be two midterms and a final exam for this class. The exam are scheduled for

Monday, October 6, 7-9pm,

Monday, November 3, 7-9pm,

Friday, December 12, 9am-12pm.

If you have a conflict with either midterm exam date, please see me **two weeks** before the scheduled exam date to arrange to take the exam early. **If you become sick shortly before the exam, please go to the health center to obtain documentation.**

Note that **vacation plans are not a legitimate reason for arranging an early final exam** so please plan to be on campus until after our exam.

Attendance and Etiquette

I expect you to attend all classes. Unless you check with me ahead of time, please arrive on time, put your devices in your bag, and stay in the classroom for the full period. This applies to exams as well as to regular class meetings. This is an active learning class, so your success in this class will depend on your engagement. Your attentive presence in class also makes a significant contribution to class as a whole.

Your class participation grade will be based on your attentive and active contribution to our class. You can miss up to three classes, for whatever reason, without penalty. Further absences will have a negative impact on your class participation grade. If you get sick and need to miss a number of classes as a result, please stay in touch and we will make a plan to accommodate this.

Honor Code

You may (in fact you are encouraged to!) work together in pairs or groups on your homework assignments. However, the final completion of each of your assignments must be your own. The best way to achieve this is to **talk with classmates** about how to decipher a problem, but **write your answer on your own**. You are expected to complete exams on your own. All exams will be closed-book and there will be no calculators required or allowed.

In a similar vein, the use of any resources beyond those given in the class (videos, notes, handouts, textbook, conversations with classmates or me) is not permitted when completing your homework assignments or exams. Our homework and exams are designed to provide you the direct practice needed to develop your mathematical skills and proficiency, as well as allow you to receive feedback that is appropriate to your particular situation so that you can learn. Since the use of AI tools (e.g. ChatGPT) goes counter to this, they not allowed in this course. Use of such tools for any assignment or exam in this course will be considered a breach of the Honor Code.

Students with Disabilities

Students who have Letters of Accommodation in this class are encouraged to contact me as early in the semester as possible to ensure that such accommodations are implemented in a timely fashion. For those without Letters of Accommodation, assistance is available to eligible students through the Disability Resource Center (DRC). Please contact ADA Coordinators Jodi Litchfield and Peter Ploegman of the DRC at ada@middlebury.edu for more information. All discussions will remain confidential.

Grading

I will determine final grades according to the following percentages:

Homework	20%
Exam 1	25%
Exam 2	25%
Final Exam	25%
Class Participation	5%

Tentative Schedule of Topics

Week Beginning	**Topics**
Sept 8	Integers, induction, equivalence relations, functions
Sept 15	Definition, examples, and properties of groups
Sept 22	Subgroups, cyclic groups
Sept 29	Cyclic groups cont, fundamental theorem of cyclic groups
Oct 6	(Exam 1) Permutation groups (Fall Break)
Oct 13	Even and odd permutations, isomorphisms
Oct 20	Cosets, Lagrange's theorem, external direct products, normal subgroups
Oct 27	Factor groups, internal direct products
Nov 3	(Exam 2) Homomorphisms
Nov 10	First isomorphism theorem, intro to rings, properties of rings
Nov 17	Subrings, integral domains, characteristic of a ring, defn of an ideal
Nov 24	(Thanksgiving break)
Dec 1	Factor rings, ring homomorphisms, first isomorphism theorem for rings
Dec 8	Wrap-up